Cosmological Difficulties with Modified Newtonian Dynamics (or: La Fin du MOND?)

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ABSTRACT
The cold dark matter paradigm has been extremely successful for explaining a wide range of cosmological phenomena. Nevertheless, since evidence for non-baryonic dark matter remains indirect, all reasonable alternatives should be explored. One proposed idea, involving a fundamental acceleration scale $a_0 \simeq 1$–$2 \times 10^{-10} \mathrm{m\,s^{-2}}$, is called MOdified Newtonian Dynamics or MOND. MOND was suggested to explain the flat rotation curves of galaxies without the need for dark matter. Whether or not it can adequately fit the available data has been debated for almost 20 years (and we summarise many of these studies), but only recently have there been studies attempting to extend MOND to larger scale regimes. We discuss how the basic properties of MOND make it at best ambiguous to apply these ideas to cosmological scales. We emphasize the difficulties inherent in developing a full theory in which to embed the main MOND concepts. Without such a theory there is no obviously consistent way to discuss the early Universe and the growth of perturbations. Recent claims that MONDian cosmology works very well are therefore not supportable. We also provide an argument for why $a_0 \sim cH_0$ naturally, a coincidence which is often suggested as a motivation for taking MOND seriously. We discuss other alternative theories of gravity concluding, as others have, that no metric theory extensions appear workable for explaining rotation curves as well as other observed phenomena. The whole premise of many of these attempts is fatally flawed – galaxies are not pre-selected, discrete, isolated regions which formed monolithically and around which one can construct an axially-symmetric dynamical model in order to remove the need for dark matter. In the modern view, galaxies are part of a dynamic continuum of objects which collectively make up the evolving large-scale structure of the universe.

Key words: gravitation – cosmology: theory – large-scale structure of Universe – cosmic microwave background – dark matter

1 INTRODUCTION

A model based on the growth of small fluctuations through gravitational instability in a universe with cold dark matter (CDM) provides an excellent fit to a wide range of observations on large scales, $\gtrsim 1\,\mathrm{Mpc}$ (see e.g. Liddle & Lyth 1993, Peacock 1998). However, the nature and properties of the CDM, apart from its being cold and dark, remain mysterious. Since there is no *direct* detection of this form of matter, one should be cautious about accepting the idea of dark matter casually, and remain open minded to other possibilities. Recently in fact a great deal of attention has been focused on some apparent failings of the CDM model on scales of galaxies (e.g. Hogan & Dalcanton 2000, Sellwood & Kosowsky 2000, Firmani et al. 2001). This examination of details of the CDM model has led to a resurgence of interest

in other concepts for modelling the dynamics of galaxies. Ideas which have been proposed to remedy these perceived problems include self-interacting dark matter, warm dark matter, fine-tuned initial conditions, and modifications to gravity. Obviously there are less exotic remedies as well, for example those depending on baryonic processes, such as gas physics, cooling and feedback mechanisms. Baryonic contributions to dynamics are in many cases expected to be non-negligible but are as of yet notoriously difficult to calculate reliably.

The focus here will be specific proposals to modify gravity, which has been the subject of much attention of late. It has been suggested that the model for the fundamental laws of physics could be changed to accommodate astrophysical observations of galaxy dynamics without the need to invoke dark matter. The best known such example is Mil-

grom's theory of Modified Newtonian Dynamics (MOND; Milgrom 1983a, 1994). The body of literature on MOND ranges from bold claims of success in every tested arena (Milgrom 1999) to discussions and identification of many of its failures (e.g. van den Bosch & Dalcanton 2000a). While it is clearly prudent to investigate all the alternatives to the dark matter hypothesis, particularly since no specific candidate particle has been detected (except perhaps a close to negligible neutrino contribution), we conclude in this paper that MOND does not appear to be promising.

Much of the focus of MOND has been on the study of galaxy dynamics and structure. We shall briefly review this work and its criticisms below. Recently, however, there has been a push to extend MOND to the cosmological realm (e.g. McGaugh 1999, 2000, Sanders 1998, 2000), and it is this aspect of MOND which is our main concern. One claim is that the low amplitude of the second acoustic peak in the CMB anisotropy data is a 'prediction' of MOND. In fact as we will discuss, this calculation is simply for a high baryon density model dominated by a cosmological constant, which also fails for other reasons. But in fact there is no obviously consistent way to carry out such calculations in a truly MONDian picture. We find several difficulties with MONDian cosmology that appear to be insurmountable. Avoiding all of these difficulties makes the MOND picture appear more and more 'epicyclic'. The existence of some form of non-luminous matter within the context of conventional physics seems, to us, considerably simpler.

We start by listing many of the motivations for dark matter in the standard model, We then enter into a discussion of the failings of baryons-only predictions for one of the main pieces of cosmological data, namely Cosmic Microwave Background (CMB) anisotropies. This is relevant, since the predictions for MOND and standard cosmology *might* coincide for this calculation (McGaugh 2000). We briefly describe MOND and then delineate its conceptual and empirical difficulties, including a review of the numerous issues previously discussed by other authors. We continue with a discussion on structure formation in these models. We then mention other alternatives which share some of the features of MOND, as well as some additional future tests which will soon be possible.

2 DARK MATTER IN THE STANDARD MODEL

2.1 Evidence for Dark Matter

Let us begin by reviewing the main evidence for large amounts of matter in the Universe which are not associated with the luminous components. There are several observations which are usually interpreted as providing evidence for (cold) dark matter, including:

- the rotation curves of galaxies compared with their light distributions;
- the gas content of clusters compared with velocity, x-ray or lensing mass estimates;
- the normalization of galaxy clustering compared with microwave anisotropies;
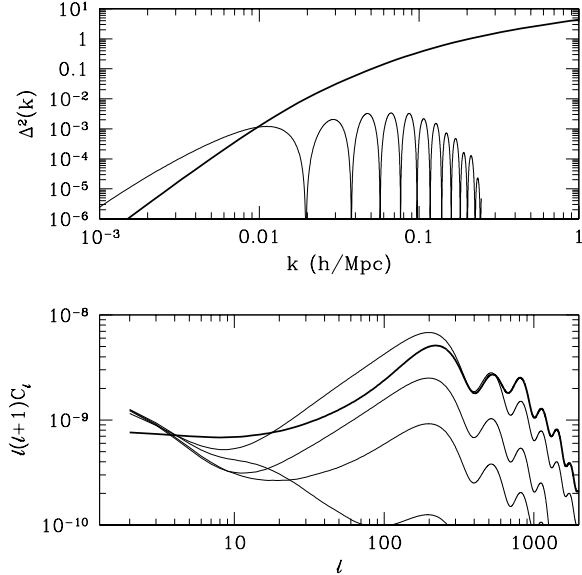- the shape of the large-scale galaxy correlations;

Figure 1. Power spectra for (matter) density perturbations (upper panel) and for CMB anisotropies (lower panel). The thicker lines are the usual ΛCDM 'concordance' model, while the other lines are the vacuum-dominated (ΛBDM) model suggested by McGaugh (2000). All models have been *COBE* normalized. In the lower panel the four Λ curves are for reionization optical depths of $\tau = 0$, 0.5, 1 and 2, with the latter perhaps the most likely for a MOND picture in which galaxies form early. The MONDian optimistic view would be that all of the shortcomings of these curves could somehow be solved in a full cosmological theory which includes modified gravity.

- the lack of strong Silk damping and existence of small-scale structure (e.g. Ly-α forest);
- cosmic flows and redshift space distortions;
- and the amplitude of weak lensing by large scale structure.

Recent reviews of several of these topics are provided by e.g. Dekel, Burstein & White (1997), Bosma (1998), Turner (1999), Peacock (2000) and Primack (2000).

This list is not exhaustive. Let us give another, more recent example, coming from CMB experiments probing the damping tail (the Cosmic Background Imager experiment in particular) which provide a further independent constraint on $\Omega_{\rm CDM}$ and $\Omega_{\rm B}$ (see White 2001 for more extensive discussion). Here $\Omega_{\rm CDM}$ and $\Omega_{\rm B}$ are the density parameters in cold dark matter and baryons, respectively, and $\Omega_{\rm M}$ is the sum of the two. The ratio of the damping scale and the spacing between the acoustic peaks, $\ell_{\rm D}/\ell_{\rm A}$, depends only on $\Omega_{\rm M}h^2$ and $\Omega_{\rm B}h^2$. It is independent of the distance to last scattering, i.e. the geometry of the Universe and any late time effects such as a cosmological constant or quintessence. The sense of the Ω dependence is that, at fixed $\Omega_{\rm B}h^2$, the lower the matter density the smaller is $\ell_{\rm D}/\ell_{\rm A}$. Phrased another way, if we require a peak at $\ell \sim 200$ then the amount of power at $\ell \sim 10^3$ decreases exponentially as we lower $\Omega_{\rm M}h^2$. Thus lower limits on C_ℓ for $\ell \sim 10^3$ can provide strong lower limits on the physical matter density of material in the Universe in an almost model independent way.

MOND represents an attempt to explain the first item in the above list without the need for dark matter, and there are claims that some of the other items can also be accommodated (see, e.g., Milgrom 1999). There is clearly a difference of opinion here among researchers about how to apply Occam's razor to the dark matter question. Some authors appear to regard as anathema the idea that some matter might not be very luminous. We suggest that it might indeed be reasonable to consider radical changes to well-known physics *if* rotation curves were the sole piece of evidence in favour of dark matter. But this is far from being the case. A baryons-only universe actually has difficulties on several fronts, as we now discuss.

2.2   A baryonic universe?

The models discussed by McGaugh (2000) and Sanders (2000) have relatively high Ω_B, and only baryonic dark matter, apart from a large contribution from Ω_Λ, to make the Universe flat or somewhat closed. These authors then appeal to possible physics within a MONDian cosmology to fix up some of the immediate problems.

Before discussing the MOND proposition itself, we will mention two consequences of having no CDM in the usual paradigm. These two particular consequences depend only on the cosmological parameters used in the MOND cosmology and not on MONDian dynamics per se; as they are more general, we mention them before tackling MOND more explicitly below.

In order to fit recent CMB data (see e.g. Pierpaoli, Scott & White 2000, de Bernardis et al. 2000, Hanany et al. 2000), a model with no CDM has to have a baryon fraction which is at least mildly challenging for nucleosynthesis constraints, and it requires a cosmological constant which is *much* larger than lensing and other constraints allow. The number of strong gravitational lenses for $\Omega_\Lambda \sim 1$ would be over 5 times the currently observed number (Kochanek 1996). Since the effective value of Ω contributed by stars and gas in known components of galaxies is estimated to be around 0.004 (Fukugita, Hogan & Peebles 1998), it is also true that McGaugh's model needs to contain about an order of magnitude more dark baryons than luminous baryons. Therefore even if particle dark matter is avoided, this is not true for dark matter in general.

It should be noted that the CMB fit itself, for the simplest case, is also poor. Fig. 1 shows that *COBE*-normalized models with only baryons (and a large cosmological constant component in order to make the Universe close to flat) has slightly too high a peak – these models were discussed in McGaugh (2000) but this difficulty was not stressed. Hence in order to use baryons alone to fit CMB data like those from the BOOMERANG experiment, it is necessary to invoke some other physical explanation for why the large angle anisotropy signal has been underestimated relative to the smaller scales. This may be possible to achieve by changing the model from the simplest case, for instance using a combination of gravity waves, tilt and reionization. Naively the modification required to make the spectrum fit are quite severe and arguably fine-tuned. In order to quantify the difficulties in utilizing reionization, more details of structure formation in the baryons-only theory are needed. Moreover,

the matter power spectrum (upper panel of Fig. 1) is disastrous for baryon-only models.

Combining the CMB and any one of a number of other cosmological constraints makes the high Ω_B solution untenable. This was stressed by Lange et al. (2000), Jaffe et al. (2000) and Bond et al. (2000) in direct analysis of BOOMERANG and Maxima data, as well as through careful multi-parameter fits by Tegmark & Zaldarriaga (2000) and others. There are two disjoint regions of parameter space that can with more or less equal capacity fit the CMB data *taken alone*. But the high Ω_B region rapidly shrinks to zero when other constraints are added. Griffiths, Melchiorri & Silk (2001) use the CMB data plus supernovae observations to conclude that no reasonable baryon-only model can fit. In fact the no-CDM part of parameter space is restricted to closed models with $\Omega_B \gtrsim 0.13$, and this is discrepant with Big Bang nucleosynthesis limits (as well as implying an amount of dark baryons almost as high as the amount of CDM required normally).

The comparison of the CMB amplitudes with a galaxy clustering normalization, such as σ_8 (the variance on $8h^{-1}$Mpc scales) also pose problems for this model. As these arguments relating the CMB to something like σ_8 rely on a number of assumptions, it is prudent to check for possible loopholes. Three possibilities certainly come to mind: (1) $n > 1$ power spectra, or power spectra with features in fortuitous places; (2) the existence of an extra dark matter component, such as hot dark matter; or (3) extra fluctuations due to an isocurvature component or topological defects. Such modifications are not going to simultaneously solve all the problems inherent in the power spectra shown in Fig. 1, however.

Since the ΛCDM model shown in Fig. 1 is a fairly good fit to all the available data, both for the matter fluctuations $\Delta^2(k)$ ($\equiv 2\pi k^3 P(k)$) and the CMB anisotropies C_ℓ, it can be seen that high Ω_B cosmologies within the standard framework are an extremely poor match to the observations. If CDM is to be avoided at all costs, then the only way to go is to be much more radical about the cosmological framework. The hope might be that, with sufficiently drastic changes, some of the shortcomings of high Ω_B models might be avoided. We now turn to MOND, one such radical modification of gravity.

3   MOND

The best known suggested modification to Newtonian gravity is usually referred to as Modified Newtonian Dynamics or MOND (Milgrom 1983a,1983b,1983c). Similar ideas for avoiding the need for dark matter been put forth by several other authors (e.g. Kuhn & Kruglyak 1987, Bekenstein 1988, Mannheim & Kazanas 1989, Liboff 1992). It is difficult to be precise about the MOND idea, because the literature does not form a coherent whole. Fundamental to the idea of MOND is that it is an 'effective' theory, playing a role similar to Kepler's laws (as stressed by Felten 1984). The proponents of MOND have yet to develop the analogue of Newtonian mechanics to explain this effective theory. The absence of a full theory seriously limits the predictive power of MOND, and leads various authors to disagree as to what the observational consequences of this revision will be.

The basic concept behind MOND is that there exists a fundamental acceleration a_0 ($\simeq 2 \times 10^{-10} \mathrm{m\,s}^{-2}$ with an uncertainty of a factor ~ 2), *below* which the actual acceleration is *larger than* the Newtonian one. This is sometimes formulated by relating the 'Newtonian' acceleration, a_N, of a test particle to the actual observed acceleration, a, through the relation (Milgrom 1983; Bekenstein & Milgrom 1984)

$$\vec{a}_N = \mu(a/a_0)\vec{a}, \tag{1}$$

where $\mu(x)$ is an interpolating function with limits

$$\mu(x) = \begin{cases} x & x \ll 1 \\ 1 & x \gg 1. \end{cases} \tag{2}$$

We shall refer to the limit $a \ll a_0$ as the 'MOND regime' and in this limit

$$a = \sqrt{a_0 \frac{F}{m}}, \tag{3}$$

which is often given as the fundamental equation of MOND. Milgrom (1983) suggests $\mu(x) = x\left(1 + x^2\right)^{-1/2}$, in which case a test particle in a spherical potential experiences an acceleration

$$a = \frac{a_N}{\sqrt{2}} \left[1 + \sqrt{1 + 4\left(\frac{a_0}{a_N}\right)^2} \right]^{1/2}. \tag{4}$$

Alternative suggestions have been made for modifying gravity, whereby a logarithmic potential is added to the usual Newtonian one. Such a formulation can also be cast in the form of Eq. (2) with

$$\mu(x) = \frac{\sqrt{1+4x} - 1}{\sqrt{1+4x} + 1} \tag{5}$$

(Kinney & Brisudova 2000). However the additive formulation is not exactly the same as 'classical' MOND, in that it specifies a fundamental *length* scale rather than a fundamental *acceleration* scale. Furthermore, the additional potential can be made proportional to the product of the masses of the interacting bodies, thus circumventing some of the problems we detail below. We now turn to conceptual difficulties with MOND and then with MONDian cosmology. We then summarize empirical difficulties over a range of scales.

3.1 Conceptual difficulties with MOND

While it seems straightforward to modify the universal law of gravitation on large scales, this is in fact not so. Newton's laws of motion and the law of gravitation are closely woven together in such a way that simple modifications rapidly lead to unpalatable consequences.

One immediate question is whether MOND applies equally to *de*celerations as to accelerations, or indeed whether the motion needs to be just a change in the vector direction of acceleration in order to show MOND effects. The usual interpretation is that all changes in velocity are subject to MOND. Immediately, we see a fundamental difference with standard dynamics when we consider a test particle moving away from a central mass. In the MONDian picture, the test particle's deceleration never drops below the value a_0. Hence it cannot escape to infinity – in MOND there are no unbound orbits. We will return later to this conceptual property of MOND, which is joined by several other

ambiguities which make it difficult to outline a consistent MOND picture when multiple systems are involved.

Several profound difficulties with Milgrom's original proposal as stated were identified by Felten (1984) soon after the introduction of the MOND prescription. One example is that MOND violates Newton's third law, since it is not symmetric between a galaxy and a test mass. Another way of stating this is that because acceleration is not inversely proportional to mass, momentum is not in general conserved for an isolated system. In addition, since the gravitational force is no longer linear, the simplest MOND incarnations have the total gravitational force not being equal to the sum of the partial forces. In particular, the motion of the centre of mass of a body no longer obeys[*] the familiar motions of undergraduate mechanics unless all of the bodies in the system have equal masses.

One can still go forward with numerical studies by implementing some prescription for determining accelerations in a given system. However, on a technical level, this property makes simulating a MOND system extremely challenging, as techniques such as N-body cannot be applied unless each body represents an actual physical component of the system under study (e.g. an individual star or perhaps even an atom). As dynamical friction within the context of MOND will be different, basic interactions such as merging will also undergo modification.

By specifying a fundamental constant with the dimensions of acceleration, MOND implicitly violates Lorentz invariance, and thus cannot be fit into our conventional theories of modern physics. Coordinate invariance, which underlies relativity, is gone, as is any concept of modelling forces as exchange particles.

Much of the challenge involved in making MOND part of a complete and consistent theory is due to the realm where it extends Newtonian theory. One often thinks about how a more complete theory gives a *stronger* effect than the simpler theory in the strong regime – but for MOND the acceleration is *larger* (than in the Newtonian theory) when it falls *below* a particular value. MOND must have this sign in order to explain rotation curves with no dark matter. However, this is the source of many of the problems at large scales.

A more specific problem is that the MONDian 'potential' tends to $\log(r)$ at large distances, as the Newtonian acceleration, given a large enough distance, will always fall below the MONDian critical value. So unless the potential is further adapted to cut it off somehow, then all objects are bound to each other; there is no such thing as escape velocity in MOND. There are cosmological extensions of MOND which address this and these will be discussed below.

The biggest theoretical dilemma comes in deciding how to interpret accelerations when multiple motions are involved: do we consider some absolute acceleration, or only peculiar accelerations? We will elaborate on this important point below. Suffice to say that this makes it hard to place MOND within a cosmological context, and suggests that the Cosmological Principle cannot really apply.

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[*] As Felten (1984) points out, this makes the original papers of Milgrom inconsistent, or ambiguous at best, since astronomical data describe multi-particle systems.

This difficulty deciding *which* acceleration one considers was already understood by Milgrom (1983a) when he proposed MOND. This arises from the problem in explaining the lack of a mass discrepancy in wide binaries (Close, Richer & Crabtree 1990) and open clusters (Leonard & Merritt 1989). Considering orbits around the Sun, MOND effects would become important for the solar system at about 7000 AU, which is much closer than the nearest stars. Hence, in loose groups of stars, one might expect to find that stellar evolutionary models for the stars would require less mass than implied dynamically by the groups. This has been avoided in Milgrom's prescription by choosing a preferred frame and considering the *absolute* acceleration in this frame. However, this would mean throwing out some fundamental physics concepts which underpin ones ideas about relative motion. The proposal is that if a group of stars is accelerating around the centre of the Galaxy, and that acceleration is larger than a_0, then there will be no MOND effects observed, even if the relative accelerations among the stars are less than a_0. In other words MOND essentially requires an absolute meaning for acceleration, and a special status for galaxy-scale collections of mass. We will come back to this when we try to understand MOND within the cosmological context in the next sub-section.

In summary, here are some of the concepts naturally implied by MOND, which would have to be tackled to make it a complete theory:

- MOND explicitly violates the equivalence principle;
- MOND violates conservation of momentum (Newton's third law);
- MOND violates Lorentz invariance;
- MOND may violate the Cosmological Principle;
- MOND does not allow for superpositions of gravitational fields;
- and MOND suggests that all bodies are bound to each other.

Thus we see that MOND is far from being simply a modification of a law in a poorly tested regime. It violates basic principles which underlie our entire framework for theoretical physics. This is a very high price to pay for explaining even the most puzzling of astronomical data! It may be that some of the obstacles listed above can be overcome if MOND could be embedded within some complete and consistent theory. However, one should not underestimate the magnitude of this task.

Although there have been attempts to derive full theories which have similarities with MOND, these have not been very successful. Bekenstein & Milgrom (1984) presented a toy model for a full theory of gravity which might contain MOND. This model gets around some of the problems described above by defining the force to satisfy $F = ma$, but at the end of the day is simply a non-relativistic potential model that falls far short of predicting the wider behaviour of such a theory. Mannheim & Kazanas (1989, see also Mannheim 1997, 2000, and references therein) attempted to derive a covariant general theory of gravity that has some features in common with MOND. Sanders (1997) described scalar-tensor theories that might accommodate MOND. However, MOND certainly implies a preferred frame and leaves details of how to calculate anything still very unclear. Typically such ideas applied to the scale

of galaxies (for example) treat the centre of the galaxy as a special point about which to perform calculations. Similar bi-metric theories have also been proposed by other authors (e.g. Drummond 2001 and references therein). None of these models appears 'natural', all suffer from further physical awkwardness (e.g. causality problems, behaviour of local gravity, stability considerations or gravitational lensing), and none provides a detailed framework in which to carry out cosmological calculations.

Periwal (1999) suggests an approach to quantum gravity involving an ultraviolet (i.e. high energy) fixed point which might have non-Newtonian dynamical effects at large scales. The basic idea is that if we assume the existence of an ultraviolet fixed point for gravity then there will exist a scale at which gravity will become a strong force. This mechanism operates in quantum chromodynamics where the force is 'weak' at short distances but becomes strong enough to confine quarks into hadrons at long distances. In analogy, gravity might exhibit qualitatively different (and perhaps stronger) behaviour at very long distances with a characteristic length scale ξ or acceleration scale $\sim 1/\xi$. To make this proposal more concrete, one would like to to compute the existence of the fixed point, the scale at which the transition takes place and the expected qualitative changes, however there is not yet currently a calculable theory of quantum gravity.

Others have suggested ideas with similar flavour to MOND, but typically these are phenomenological models only, for example introducing a special scale rather than an acceleration, or modifying G. None of the suggestions that we are aware of help with the issue of how the concept behind MOND might be embedded in a genuine theory.

3.2 Difficulties with MONDian cosmology

To say that we have no idea how the early Universe works in a MONDian picture would be a gross understatement. The fact that MOND is not relativistic makes it difficult to interpret the scale factor and the Hubble law, and the possible acceleration of the Universe. In order to perform cosmological calculations, one needs some way of generalizing the theory.

To describe cosmology, we will assume that we start with homogeneous and isotropic initial conditions. It is not obvious how to produce scale invariant initial conditions such as would arise from inflation in such a theory, however we will focus on consequences from MOND for subsequent structure formation here. Specifically, we imagine that somehow a theory can be found which at early times results in a universe that is, to a good approximation, homogeneous and isotropic. Since we must be careful not to make relativistic statements in this theory, we shall assume that this holds true in the 'absolute' rest frame of the Universe (we shall return to this below).

We then need to include the Hubble expansion and perturbations. The most explicit work on cosmology within MOND is by Sanders (1998, 2000), using ideas explored earlier by Felten (1984), and we will describe their generalizations below. The possible cosmological prescriptions differ in their applications of the original MOND equation, $\mu(a/a_0)a = a_N$, distinguished by which accelerations are used in which place on the left hand side. Separation is made

into a 'peculiar' acceleration a_P and a 'Hubble' acceleration a_H. The Hubble acceleration is taken from the Friedman equation

$$\frac{\ddot{R}}{R} = -4\pi G(\rho + 3p), \tag{6}$$

with a point at position $r = R(t)r_0$ having acceleration $\ddot{r} = r\ddot{R}(t)/R(t)r$. Note that the acceleration thus depends on the reference point used to determine r (that is, r_0).

Requiring the MOND prescription to affect a_P at the very least, in order to reproduce effects in galaxies, we have the following four possibilities:

(i) $\mu((a_H + a_P)/a_0)(a_P + a_H) = a_N$;
(ii) $\mu(a_P/a_0)a_P = a_N$;
(iii) $\mu(a_H/a_0)a_P = a_N$;
(iv) $\mu((a_H + a_P)/a_0)a_P = a_N$.

One could also imagine a_0 varying with cosmological time, a suggestion which has already been made (e.g. Sanders 1998), and which we do not consider further (but see §4). We reiterate that that *both* accelerations a_P and a_H implicitly assume a specific frame of reference and thus further specification is needed to complete these definitions.

Now let us consider each of these four possibilities in turn.

It is simplest to consider the first option (i) for the case of pure Hubble expansion (i.e. set $a_P = 0$, which will be true at sufficiently early times). Choosing the 'absolute' rest frame as described above, with this prescription, initially the total acceleration on any fluid element tends to zero (the MOND regime) because of the homogeneity and isotropy. The vector sums of all of the accelerations tend to zero simply using spherical symmetry about any point. Note that this argument is completely independent of the functional form of the force law, it depends only upon the symmetries of the problem and that the force acts along the line joining the two bodies (so as to have for example Kepler's equal area law for elliptical orbits).

This is the case considered by Felten (1984), in analogy with the McCrea & Milne (1934) approach to deriving the Friedmann equations from Newtonian dynamics. As noticed immediately by Felten (1984), the 'cosmological equations' of MOND do not admit a homogeneous and isotropic universe that obeys the cosmological principle. As distant objects in MOND remain bound to each other with an approximately logarithmic potential, perturbations in the Universe will presumably[†] always collapse – every system will have negative total energy. Assuming a central point can be chosen somehow, regions collapse quickly, out to scales of perhaps 30 Mpc in the present universe (Sanders 2000). For this reason, Sanders makes the assumption that MOND only applies to determining the peculiar accelerations. This allows the next three prescriptions.

We note that another way to violate this argument is to suggest that the frame in which the Universe is isotropic and homogeneous is in fact accelerating with respect to some fundamental frame against which we measure accelerations

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[†] Since it is at present impossible to formulate a self-consistent cosmology, it is not possible to prove that perturbations will always collapse.

in the MOND universe. If this is true then there is *no* scale on which MOND effects operate. We could turn MOND back on by having the entire Universe decelerate with respect to the fundamental frame at a later time, in effect 'turning on' the MOND force. Obviously this mechanism can allow us to turn MOND on and off as many times and at whatever points we desire. Note however that in the transition regime, where the acceleration of the Universe with respect to our background frame is $\mathcal{O}(a_0)$, there will be a preferred direction to the force law. Objects whose matter induced acceleration is oppositely directed to our fundamental acceleration will have more 'MOND acceleration' than those which are accelerating in the same direction. The existence of a preferred direction would be quite an unpleasant side-effect of this mechanism.

Let us now turn to case (ii) from the above list, As soon as perturbations are introduced into our early Universe, MOND will kick in with full force and the evolution will be distinctly different than in the conventional theory. Hence CMB anisotropy calculations using CMBFAST (Seljak & Zaldarriaga 1996) for example, are not valid. And, in general, there is no way to get anything like the usual cosmological results in this case.

For (iii), the opposite problem occurs. Again, a central point for the Hubble flow must be chosen and any system far enough away from it will again show no effects of MOND. Thus MOND would only apply within a certain distance of some specified region.

Option (iv) is the one advocated by Sanders (2000). To go further, one has to decide how to define the peculiar and Hubble accelerations. The first option is to choose one point and to define these accelerations from this position. This quickly has the same problem as suggestion (iii). That is, we take as a physical model, for example, that the Universe is empty and large, and into this universe explodes an expanding fireball containing all of the matter and radiation in what we think of as our Universe. The fireball cools and the expansion (rapidly) slows due to self-gravity. It is this overall acceleration that stops us from feeling the MOND force. We fortunately live at the centre of this expanding fireball which has now cooled to the point where we are matter-, and not radiation-dominated. Thus the McCrea analogy for deriving the Friedmann equations must in fact represent the true physical situation and not be a convenient pedagogical way of invoking Birkhoff's theorem.

However, if one proposes an absolute meaning for acceleration, then a distant galaxy or cluster has a large a_H and so its internal kinematics would be Newtonian not MONDian, and hence this would not explain rotation curves, etc. The key point is that you cannot imagine moving to the rest frame of that distant galaxy (which would allow it to have a flat rotation curves) because you have given an absolute meaning to acceleration in MOND.

A second way to address this appears to be the intention of Sanders (2000) – one considers a shell which has broken away from the general expansion. In this case, a_H and a_P are defined with respect to the centre of this collapsing shell. This means assuming an isolated overdensity, insensitive to other possible sources of peculiar velocities, i.e. other inhomogeneities outside of it. This seems hard to reconcile with the modern cosmological view. In the conventional picture, velocities are driven by perturbations over a range of

scales, and tidal forces generate galaxy spins, for example. Nevertheless, if we assume this is somehow true, we still run into difficulties. At the same time as ignoring forces from external perturbations, the background cosmology can affect the expansion of this shell – in fact it decides the transition between the Newtonian and the MONDian regime.

It becomes awkward to try to apply this rule to each shell in the specified region, as accelerations cannot be separated in MOND and thus another ambiguity arises. One possibility is that one applies this rule to the largest such shell in each region. Thus, the Hubble acceleration for scales in the shell is taken to be whatever the Hubble acceleration is at that scale measured from the centre of the largest collapsed shell containing the system. At the present time, the largest such shell would be approximately cluster-sized and so perhaps MOND should not be used for any galaxies on the outskirts of a cluster?

A problem with this definition is that a region will go from the Hubble flow to the MOND region and then perhaps out of the MOND region all depending on its distance from some central point. The definition of this central point will often change in the process of merging and collapse of larger scales in what appears to be an instantaneous non-local manner. Nevertheless, this prescription seems closest to what is suggested in Sanders (2000). However the ambiguities involved make it difficult to make any definitive statements about how to treat cosmological perturbations within MOND.

In addition, the problem above, that the Hubble acceleration can be $\leq a_0$, will also have been expected to occur in our history as the Universe is now accelerating but was at one time (in both the concordance cosmology and e.g. that suggested by McGaugh 2000) decelerating (we return to this in §3.4).

If we proceed with the calculation using the Hubble flow as part of the acceleration for purposes of invoking MOND (option (iv) above), then there is an ambiguity about *which* scale we are supposed to use. In other words, the outer regions of some galaxy could define the relevant shell and then we would say that MOND should be used for that particular galaxy. But we could also look at a nearby galaxy and say that it is part of a bigger shell, in which case we do not use MOND. How one would consistently treat a set of neighbouring merging shells, destined to become a filament for example, is entirely unclear.

Sanders (2000) deals with the case of one collapsing shell, and he agrees this prescription as it stands is a first approximation and possibly not self-consistent. The growth of non-linear structure, which involves collapsing objects within other collapsing objects and their collisions, is thus difficult to calculate reliably. These inherent ambiguities, together with the smallness of the Hubble acceleration, and the intrinsic non-locality of this picture, make option (iv) seem quite unworkable.

So where have we come in this discussion? If it has appeared less than crystal clear, then it is because MOND is both ambiguous and not obviously self-consistent when it comes to extending the idea beyond individual galaxies to the cosmological context. There are several possibilities for *which* acceleration one should applies the MOND equation. We have tried to consider each possibility in turn. The conclusion is that it is difficult to find any consistent framework

in which calculations could be attempted. It seems that the only way to have MOND make sense is to arrange for the centres of galaxies to be pre-selected as special places around which to consider MONDian gravity. Perhaps this would be reasonable in a picture where galaxies were discrete, isolated and formed monolithically. However, it is very hard to reconcile this with the Universe as presently understood, containing fluctuations over a wide range of scales, as well as merging hierarchical clustering and the rest.

3.3   Empirical difficulties with MOND

Some authors have indicated that MOND has been very successful in explaining observations of rotation curves for a variety of objects over a wide range of scales (see e.g., Milgrom 1999). We wish to stress that a number of studies have indicated difficulties in reconciling MOND with data under the assumption that there is *no* dark matter, and that the scale a_0 has a fixed value. Dressler & Lecar (1983, cited in Felten 1984) appear to have been the first to object that MOND did not adequately fit their data, but they did not publish this. Subsequently, a large body of published work has accumulated over the years. Since this corpus is not referred to very much in the MONDian literature, let us try to be fairly comprehensive here.

Kent (1987) pointed out that although MOND could fit his H I rotation curve data there was a factor of 5 range in the value of a_0 required and also no clear evidence for the slightly falling rotation curves that MOND would still predict. Hernquist & Quinn (1987) examined simulations of shell galaxies within MOND, concluding that the observed number and radial distribution of shells in NGC 3923 could not be explained without a dark matter halo. The & White (1988) found that a MOND fit to the Coma cluster requires a higher value of a_0 than for galaxies and also does not predict the correct temperature profile for the x-ray emitting gas. Lake (1989) pointed out discrepancies between MOND and observations of dwarf irregular galaxies. In particular Lake & Skillman (1989) found that MONDian fits to the Local Group dwarf IC 1613 would require values of a_0 at least an order of magnitude below the favoured values. Kuijken & Gilmore (1989) also considered MOND in their study of the distribution and dynamics of K dwarfs. They concluded that the vertical accelerations in the solar neighbourhood require the presence of a dark halo and are quite severely inconsistent with MOND. Gerhard & Spergel (1992) investigated dwarf spheroidal galaxies in the Local Group and found that some of the dwarfs need to contain some dark matter even under the MOND hypothesis. Gerbal et al. (1992, 1993) compared data on several x-ray clusters with MOND, finding that in several cases the MOND fits suggested less mass than implied by the x-ray emitting gas alone. In addition MOND predicted too strong a concentration of the mass towards the centre, and could not in general fit the data without requiring dark matter. Lo, Sargent & Young (1993) described neutral hydrogen data for dwarf irregular galaxies which would have a mass below the observed H I mass under the MOND hypothesis. Christodoulou, Tohline & Steiman-Cameron (1993) studied tilted ring models for the H I distribution in NGC5033 and NGC5055. They found evidence for prolate potential wells, which would be difficult to accommodate in MOND. Buote & Canizares (1994) discussed the

ellipticities of the x-ray vs optical isophotes of the elliptical galaxy NGC 720 showing that MOND cannot explain the flattening of the x-ray isophotes without requiring a component of dark matter. Soares (1996) found that MOND can only explain the velocity distributions of binary galaxies if either the mass to light ratios are high (i.e. there is lots of dark matter) or the galaxies are on very eccentric orbits, and in that case the distribution of separations is not consistent. Sánchez-Salcedo & Hidalgo-Gámez (1999) looked at disc instabilities of gas-rich dwarfs within MOND and concluded that these would be catastrophic. Giraud (2000) discussed how variations in mass-to-light ratios between galaxies in the standard dark matter picture are more palatable than variations of a_0 within MOND. Blais-Ouellette, Amram & Carignan (2000) reported problems fitting the detailed kinematics of the well-studied late type galaxy NGC 3109 within MOND. Additionally they found that fits to late type galaxies appear to require higher values of a_0 than for early type galaxies. Bothun et al. (2000) investigated a new sample of Cepheids in the large spiral NGC 2841. MOND could give an acceptable fit to the rotation curve of this galaxy, but only if it lies considerably further away than its Cepheid-implied distance. van den Bosch & Dalcanton (2000a, 2000b) have carried out a very detailed study of dwarf rotation curves in the MOND picture. They concluded that MOND cannot reproduce both the Tully Fisher relation and the lack of high surface brightness dwarf galaxies. Dalcanton & Bernstein (2000) have continued the detailed study of galaxy rotation curve data. They found that MOND fits to low surface brightness galaxies require mass-to-light ratios which are too small to be consistent with stellar population models.

Detailed dynamical investigations of galaxies which argue against maximal discs (e.g. Courteau & Rix 1999) are by inference also arguing against MOND. In addition, studies which require haloes to be much less flattened than the luminous distribution will also tend to be inconsistent with MOND. Two recent examples of such studies are the persistence of tidal streams in the Milky Way halo (Ibata et al. (2001) and the statistics of two- and four-image gravitational lenses (Rusin & Tegmark 2001; though other factors could also be at work in this analysis).

3.4 MOND at large scales

The most recently discussed MONDian cosmological model (McGaugh 2000) is a Friedmann-like universe with a large cosmological constant ($\Omega_B \sim 0.04$, $\Omega_\Lambda \sim 1$). We have already discussed (in §2.2) some of the consequences for those scales where MOND is not operative and where one can apply constraints on the MONDian choices of cosmological parameters. We now turn to the MONDian cosmological consequences, beginning with a picture closest to the conventional cosmology, and then introducing the extensions described by Sanders (2000), assuming that somehow a consistent framework could be developed. Recall that, in this case, the expansion of the Universe contributes an 'absolute' acceleration that must be considered when evaluating MONDian forces.

In the standard cosmology, the Friedmann equations in a flat Universe give the acceleration of the scale factor as

$$\frac{\ddot{R}}{R} = H_0^2 \left(\Omega_\Lambda - \frac{1}{2}\Omega_M \right). \tag{7}$$

We shall follow McGaugh (2000) and assume this is the acceleration in a MONDian universe also.

Let us first consider an object at rest in comoving coordinates, neglecting look-back time effects. We then have for an object at distance d,

$$a = 3.24 \times 10^{-13} \left(\frac{d}{1\,\text{Mpc}} \right) h^2 \left(\Omega_\Lambda - \frac{1}{2}\Omega_M \right) \text{m s}^{-2}. \tag{8}$$

If we use interpretation (iv) and choose one reference point, difficulties arise immediately. We list these and then go to the modification (described above) implemented by Sanders (2000). With one reference point for accelerations, for the standard ΛCDM model, we find that the MOND acceleration is reached at a distance of approximately $600\,h^{-2}$Mpc from us. In the close to flat, cold dark matter-less ΛBDM models proposed by McGaugh (2000), this value is more like $400\,h^{-2}$Mpc. This means that galaxies at recession velocities $\gtrsim 60\,000\,\text{km s}^{-1}$ should cease to show MOND effects. Hence any galaxy or cluster at $z \gtrsim 0.2$ should show no behaviour normally taken to imply a dark halo!

As an example, let us consider the galaxy cluster MS 1137, at $z = 0.783$ (Luppino & Gioia 1995). This is the second highest redshift cluster in the EMSS and has an x-ray flux of $1.90 \times 10^{37} h^{-2}$ W in the 0.3–3.5 keV band. The x-ray temperature is $5.7^{+2.1}_{-1.1}$ keV. This cluster has been studied by Donahue et al. (1999), who find that the gas mass within $0.5 h^{-1}$Mpc is $1.2^{+0.2}_{-0.3} \times 10^{13} h^{-5/2} M_\odot$. Using hydrostatic equilibrium and Newtonian mechanics (supposedly valid at these high redshifts), they infer a total mass $2.1^{+1.5}_{-0.8} \times 10^{14} h^{-1} M_\odot$ (within the same region). This leads to a gas fraction $f_{\text{gas}} = 0.06 \pm 0.04 h^{-3/2}$, whereas the MOND prediction would be $f_{\text{gas}} \equiv 1$.

There are certainly other examples of $z > 0.2$ galaxies that do not appear to behave any differently than their lower z cousins. Vogt et al. (1997) has at least one galaxy with a flat rotation curve at such a redshift. Other work on the fundamental plane etc. in the most distant clusters would be hard to explain if the outer regions of galaxies slowed down considerably as MOND switched off. Wilson et al. (2001) studied statistical lensing around galaxies at $z = 0.1$–0.9. Their results are consistent with what is expected from flat rotation curves out to ~ 100 kpc, with no sign of a redshift dependence. This is hard to reconcile with a MOND picture in which the accelerations go from the MOND to non-MOND regimes over this redshift interval.

However, Sanders (2000) suggested a different prescription for cosmological accelerations in MOND. The above objection is avoided if one does *not* assume an absolute meaning to Hubble accelerations relative to us. Let us put aside the conceptual difficulties this leads to (that we already discussed). In this case, every observer's Hubble acceleration still changes with time:

$$\frac{\ddot{R}}{R} = H_0^2 \left(\Omega_\Lambda - \frac{1}{2}\Omega_M R^{-3}(t) \right). \tag{9}$$

Thus at some time in the past ($z \sim 0.3$ for a concordance cosmology and $z \sim 3$ for the parameters suggested by McGaugh 2000), the Hubble acceleration went through zero and hence cannot compensate for difficulties arising from

peculiar accelerations alone. At this time, all the MONDian effects would come into force, for a period of time determined by the size and other accelerations acting in the region considered. Sanders (1998) discusses something similar this when considering a different MONDian prescription. At best we can say that when we consider accelerations on cosmological scales the predictions of MOND are ambiguous.

3.5 Gravitational Lensing in MOND

Another entirely different constraint on MOND comes from considerations of gravitational lensing. The argument was discussed in Walker (1994), but it is so nice that we repeat a modified version here. (Also see the recent papers by Mortlock & Turner 2001a, 2001b). Similar arguments were also discussed by Edery (1999) and Bekenstein, Milgrom & Sanders (2000).

Kinney's (2000) argument about gravitational lensing of clusters of galaxies shows that all modifications of gravity have to obey the equivalence principle, in the sense that the dynamical mass must be the same as the mass which couples to photons in lensing. Once this is known, then we can treat the theory as a metric theory: the objects involved know how to move independent of their mass, and thus the 'gravity' they feel can be written as a property of space-time independent of the mass of the probe. Walker (1994) argues that lensing effects at large distances can prove disastrous for any theory in which potentials are not close to Newtonian at the largest scales as well as locally. His analysis shows that a gravitational force which falls off like $\log(r)$ is inconsistent with the mean convergence in the Universe being small. With that much field at large distances, the typical light ray gets strongly lensed, and objects are either grossly magnified or demagnified. Walker has estimated that the potential must become $1/r$ again beyond a radius of about 10^{23}m. He notes that an analysis of any particular gravitational lens may put even stronger constraints on the large-r behaviour of gravity.

Whatever mimics the dark matter has to couple to light the same way as to matter, or else you do not get the masses of clusters correct (the factor of '2' in the bending of light is the least of our worries!). Then we can show that, generically, both the convergence and shear of a lens scale as a/r where $a(\vec{r})$ is the bend angle at $\vec{r}$ and $\vec{r}$ is a 2D vector in the plane of the sky. We are implicitly assuming here that da/dr and a/r are of the same order, which should be true generally. Now for a compact mass distribution in normal GR, the bend angle falls off asymptotically as $1/r$, so our effect falls as $1/r^2$. For a distribution of sources, we need this fall-off for the total (integrated over d^2r) to be a convergent function (remember the signs of the deflections are random so a logarithmic divergence is really convergent). If we have gravity falling off more slowly than $1/r$, then the bend angle (which is just the line integral of $\nabla\Phi$ and so dimensionally goes with the same power of r as does Φ) also falls off more slowly than $1/r$ and therefore the convergence (or shear) falls off more slowly than $1/r^2$ and the integral over 2D space is divergent.

Hence, a theory with potentials falling off as $\log(r)$ will have huge lensing optical depth, and rays will have chaotic paths through the distant outer regions of galaxies. This argument does not apply to the simplest MOND picture, since

that is not a metric theory. It has been suggested that there is some non-GR full theory, for MOND is just an approximation on the scales of galaxies. However, for even that to work, this full theory would have to mimic GR closely on both small and large scales. While clearly not impossible, this does not seem very promising. But, without anything like a full theoretical framework, it is impossible to make definitive conclusions.

4 THE VALUE OF a_0

The value of the characteristic MOND acceleration scale a_0 is the same order of magnitude as the dimensional acceleration cH_0. It has been argued that there may be some fundamental reason behind this, and that this provides a motivation for the notion that there is some deep hidden truth underlying MOND. It has also been suggested that this might lead one to consider models in which a_0 varies over cosmological time (e.g. Milgrom 1994). If the Universe is currently accelerating, then H is smaller at high z, and so high redshift galaxies would be expected to show less MOND effects if $a_0 \propto H(z)$. Taking the value today, if $H_0 \simeq 70\,\mathrm{km\,s^{-1}Mpc^{-1}}$, then $cH_0 \simeq 3.5a_0$, which is close, but not strikingly so.

From the point of view of the standard cosmological picture, this might be thought of as merely a coincidence. But as we shall argue, it is almost inevitable that a_0 has this order of magnitude. With some consideration of dimensionless numbers and a little nod to the anthropic principle, the coincidence is easily explained.

First notice that MOND was designed to fit galactic rotation curves, so that $a_0 \sim v^2/\mathcal{R}$ for the scale of a typical galaxy, $\mathcal{R}$. Now notice that the dimensionless amplitude of potential perturbations (one of the 6 fundamental cosmological numbers of Rees 1999) is given by

$$Q \sim \frac{GM}{\mathcal{R}c^2} \sim \frac{v^2}{c^2} \sim 10^{-5} \tag{10}$$

over a wide range of cosmological scales. Thus for MOND to fit galaxies we need to have $a_0 \sim Qc^2/\mathcal{R}$. Then the MOND-cosmology coincidence, $a_0 \sim cH_0$ implies that $\mathcal{R} \sim Qc/H_0$, or in other words

$$\frac{\mathcal{R}_{\mathrm{galaxy}}}{\mathcal{R}_{\mathrm{Hubble}}} \sim Q. \tag{11}$$

To rephrase this, the mystery comes down to understanding why galaxies are roughly 10^{-5} times the Hubble length.

Here we can appeal to arguments involving dimensionless numbers for setting the sizes of stars and of galaxies (see e.g. Padmanabhan 1996, §1.17 and 1.19). The consideration that a galaxy must be able to cool in a gravitational collapse time implies that

$$\mathcal{R}_{\mathrm{galaxy}} \sim \alpha_{\mathrm{G}}^{-1}\alpha^3 \left(\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}\right)^{1/2} \frac{\hbar}{m_{\mathrm{e}}c}, \tag{12}$$

where α is the usual fine-structure constant and $\alpha_{\mathrm{G}} = Gm_{\mathrm{p}}^2/\hbar c$ is the 'fine-structure constant' for gravity. A rather mild version of the anthropic principle suggests that the age of the Universe today should not be too different from the characteristic ages of stars, $t_* \sim \epsilon M_*c^2/L_*$, where M_* and L_* are characteristic masses and luminosities, and ϵ

is the efficiency of nuclear energy generation. M_* can be estimated using a well known argument that the temperature of a cloud of gas should be high enough to allow nuclear fusion. L_* can then be estimated by assuming that the photon opacity is dominated by Thomson scattering (for simplicity). An estimate for the Hubble length is therefore given by ct_*, which is

$$\mathcal{R}_{\mathrm{Hubble}} \sim \alpha_{\mathrm{G}}^{-1} \alpha^{-1} \epsilon \eta^{-3/2} \left(\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} \right)^{1/2} \frac{\hbar}{m_{\mathrm{p}} c}, \qquad (13)$$

where η is a correction factor for the nuclear tunnelling probability (see Padmanabhan 1996 for details).

We might therefore expect

$$\frac{\mathcal{R}_{\mathrm{galaxy}}}{\mathcal{R}_{\mathrm{Hubble}}} \sim \alpha^4 \epsilon^{-1} \eta^{3/2} \left(\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}} \right). \qquad (14)$$

If we use typical values of $\epsilon \sim 0.01$ and $\eta \sim 0.1$, we find that this ratio is indeed around 10^{-5}, which is the value of the dimensionless potential Q. Within the standard cosmological framework this is just a numerical coincidence, but a perfectly understandable one, without any need to invoke MOND.

5   GROWTH IN HIGH Ω_{B} MOND MODELS

Now let us look in more detail at cosmological growth of perturbations and the evolution of the power spectrum in a MOND picture. Fig. 2 shows a particular example of evolution of a range of scales in a MOND model, following the arguments presented by Sanders (2000), which, as we have discussed, are far from clear. Let us analyse Sanders' (2000) results, since the final claim is that the calculated MONDian power spectrum has a general shape similar to the ΛCDM one, and therefore can be considered in good agreement with the data.

To re-cap, the idea is that in the early Universe the relevant accelerations were larger, and hence MOND effects are negligible. Whether this is true or not depends, of course, on deciding which accelerations to choose, as well as on having a consistent cosmological framework in which to discuss cosmology at all. For now we will assume that such difficulties will one day be solved, and that the early Universe will behave as in the conventional cosmology, so that we can consider the approach Sanders (2000) has taken. Here, the fluctuations would evolve according to standard Newtonian dynamics up to the point at which the Hubble acceleration becomes small enough to trigger the MOND regime. For scales between ~ 1 and ~ 100 Mpc, this happens in the matter dominated regime. When MOND starts playing a role, the evolution of an overdensity follows a non–linear equation. Note that, for smaller scales, this 'switching redshift' would happen in the radiation-dominated regime, and so more thorough calculations would have to be done – but this is a minor point.

One issue omitted from Sanders (2000) discussion is the following: does the peculiar acceleration ever become big enough to re-establish Newtonian dynamics? Again we return to the whole ambiguous issue of the choice of acceleration. In any case we have numerically checked that under the pseudo-Newtonian calculation of the MONDian evolution of a sphere (assuming Sanders 2000 assumption about
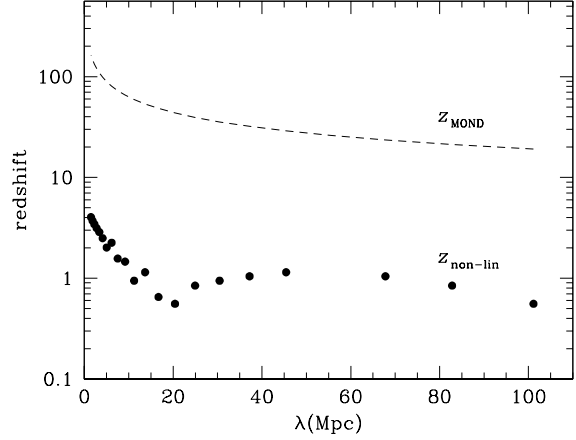

Figure 2. Redshift at which a scale might enter the MOND regime (z_{MOND}), and redshift at which it might become non-linear ($z_{\mathrm{non-lin}}$), according to the MOND-adapted spherical top-hat ansatz suggested by Sanders (2000). We have carried this out for initial conditions for a particular ΛBDM cosmology (hence the oscillations in $z_{\mathrm{non-lin}}$), and with $h = 0.7$. Although there are a number of serious problems with this calculation, if taken at face value it appears to show that in MOND even scales ~ 100 Mpc will have gone non-linear today, in stark contradiction to many separate data-sets.

the correct acceleration to use), structures on 70–80 Mpc scales would become non–linear at $z \simeq 1$ (see Fig. 2).

In general then, under this prescription we would expect structures to collapse very much earlier than in the standard cosmology. Star and/or quasar formation would thus make the Universe highly ionized at high redshift, erasing degree-scale CMB anisotropies, and possibly violating the CMB spectrum y-distortion constraint. In Fig. 1 then, the best estimate for the CMB anisotropy spectrum may be one with significant reionization optical depth (we show models with $\tau = 0$, 0.5, 1 and 2). Such a spectrum not only has a lower second peak than expected, but no first peak to speak of either!

Moreover, for the matter perturbations, if we take the modified top-hat calculations at face value, we find that scales as large as ~ 100 Mpc may have already gone non-linear (Fig. 2) today. This is entirely at odds with what we know about large-scale power, where the perturbations have *low amplitude* and appear to have close to *Gaussian statistics*. How this could be arranged if the MOND perturbations have already gone non-linear is hard to understand. So the Sanders (2000) ansatz would seem to generate massively *too much* power at large scales.

Now let us turn to the power spectrum shape. The first question would be how one even talks about a power spectrum in a framework which is inherently non-linear. This non-linearity means that k-modes are not independent in MONDian cosmology, and hence it is not sufficient to evolve a range of different k-modes in order to evolve the power spectrum. Let us imagine, however, that MONDian cosmology remains sufficiently linear until the overdensities become themselves non-linear. In that case the first thing to compute is simply the baryon-only model power spectrum.

Fig. 1 shows that the power spectrum produced by a high Ω_B model is extremely different from that produced by models with a significant proportion of CDM, if we stick to Newtonian dynamics. In addition to the total lack of small scale power, the high Ω_B model power spectra have enormous oscillations, going out to very large scales. Certainly there are plenty of data on relevant scales (e.g. Peacock & Dodds 1996, Lin et al. 1996, Tadros & Efstathiou 1996, Sutherland et al. 1999, Efstathiou & Moody 2000, Schuecker et al. 2001) that show no indication of such dramatic oscillations. While it is feasible (although not currently calculable in detail, since there is no detailed theoretical framework in which to do so) that the effects of MOND could induce new fluctuations on small scales, these would have to contrive to fill in the oscillations in the power spectrum. This seems a very tall order, particularly on such large scales. Since current data match the popular ΛCDM power spectrum rather well, it is hard to see how any more detailed MOND-based calculation would rescue these wildly different baryon-dominated power spectra.

Sanders (2000) suggests a recipe for computing a MOND power spectrum. Since the MOND evolution equation is non-linear, he considers the evolution of an overdensity in real space, rather than the more standard Fourier components of it. The scale of the spherical region in real space is identified with a wavenumber $\sim 1/k$. How precisely one interprets a set of spherical collapsing galaxies with a random field of perturbations is unclear. In computing the power spectrum, Sanders (2000) actually evolves $\Delta(k) = \sqrt{2\pi k^3 P(k)}$, which may not amount to the same result. The final power spectrum is found reversing the same formula, after evolving $\Delta(k)$ up to $z = 0$. While the lack of small scale power is partially rescued, it is still true that the imprint of the purely baryonic nature of the matter is manifest through wild oscillations in the power spectrum. In addition, although Sanders (2000) finds that the resulting power spectrum qualitatively recovers the small scale structure, the calculation is effectively modified from linear theory, and hence does not contain the full non-linear growth. Since fluctuations go non-linear much earlier in the MOND calculation (see Fig. 2), the final power spectrum would be considerably higher than claimed, and hence in qualitatively poor agreement with data.

As an aside, there is an additional level of ambiguity in the way the super-horizon scales are evaluated. This implies an uncertainty on the overall normalization of the power spectrum, and therefore on the claimed value of σ_8.

Is there any way out for a model like MOND? One possibility would be to argue that complexities due to biasing or other non-linear effects could make the measured power spectrum much smoother. However, in the non-linear regime MOND→GR and so this cannot save things; it would be necessary to get rid of these oscillations in the linear regime. To linear order you *cannot* couple Fourier modes, since differential operators become linear in Fourier space. Thus mode-coupling cannot wash out these oscillations. Simple experiments we have done with bias also indicate it is difficult to wash out these oscillations. For any power-law scheme where $\rho_{gal} \propto \rho_{mass}^B$, the oscillations survive in the galaxy power spectrum.

The only thing left then is to have growth rates which magically smooth the spectrum out, i.e. the peaks have to know to grow more slowly than the troughs. But the only new ingredient in the problem is a_0, otherwise the standard Jeans growth analysis holds. Since a_0 knows nothing about the oscillation scale for the matter power spectrum, it is unclear how it could smooth things out. Or phrased another way, there can be at most a one dimensional combination of parameters for which this is even conceivably true. Finally, it is hard to see how initial conditions could have contrived to know that these oscillations would be frozen in at $\sim 1/3$eV by the recombination of Hydrogen.

The bottom-line is that there is no legitimate way to carry out calculations of the evolution of the power spectrum within MOND. It is not that the calculations are complicated, but that there is no framework in which to carry out the calculations. Hence any claims that MONDian cosmology provides a good fit to CMB data or galaxy clustering are not currently supportable. As we have discussed, the opposite conclusion could just as easily be reached. The power on ~ 100 Mpc scales is well in the linear regime and is comfortably fit by *COBE*-normalized ΛCDM models. It seems unreasonable to appeal to an argument that the calculations cannot currently by carried out, so maybe MOND will eventually work well. The simplest picture (ΛBDM) gives disastrously low power on small scales, and oscillations on large scales, while the modified spherical top-hat calculations indicate far too much power on all relevant scales.

6 ALTERNATIVES TO MOND

There have been several earlier ideas which share some similarities with MOND. We feel it is important to realise that there have been a great many other suggestions for fitting flat rotation curves while avoiding dark matter. Most of these were abandoned quite early on or never, in fact, taken as serious alternatives in the first place.

One of the earliest such proposals appears to have been by Finzi (1963), who considered a law of gravity which becomes stronger than Newton's law beyond some characteristic length scale. Tohline (1983) discussed the possibility of the long-range force between stars in galaxies becoming $1/r$. Sanders (1984) considered an effective anti-gravity force operating on scales smaller than galaxies. And Sanders (1986) later considered a model which is like MOND but returns to $1/r^2$ with a larger value of G at scales much greater than that of individual galaxies. Bekenstein (1988, see also Sanders 1989) suggested the addition to GR of a complex scalar field whose phase couples to ordinary matter, which gives force laws which behave similarly to MOND. Meanwhile Talmadge et al. (1988) set stringent limits on deviations from $1/r^2$ force laws on solar system scales, and McFarland (1990) placed similar constraints on $1/r^2$ deviations on cluster scales. Mannheim & Kazanas (1989) proposed a covariant theory which effectively has an extra constant force at large radii. Fahr (1990) suggested an additional inductive-type term in the gravitational force due to mass currents. Goldman et al. (1992, see also Bertolami & García-Bellido 1996) proposed a particular form of variable G that might explain flat rotation curves. Battaner et al. (1992) discussed the possibility that magnetic fields could cause flat rotation curves without the need for dark matter. Gessner (1992) suggested that a large negative cosmological constant could

make rotation curves flat. Eckhardt (1993) considered a two-parameter exponential potential. Stubbs (1993) suggested the possibility of an exotic coupling between dark matter particles and baryons, and set about constraining such a coupling. Milgrom (1994) also suggested a model where dynamics might change below some characteristic frequency rather than acceleration. Soleng (1995) suggested that a point mass in a cloud of strings could give an effectively $1/r$ force law. Carlson & Lowenstein (1996) considered the addition of a constant potential term, which they claimed is motivated by conformal gravity. Drummond (2001) suggested a bi-metric theory of gravity with a scale built in at the size of galaxies, above which the effective value of G is larger.

This is not a complete list, but shows some of the range of possibilities that have been discussed. In general, the concepts are phenomenological and consider the solution of the dark matter 'problem' to be confined to explaining the haloes of individual galaxy-scale objects. Most of these ideas must surely suffer from many of the shortcomings of MOND, particularly the large scale behaviour, which may be even more pathological in some cases.

Better motivated ideas of modifying gravity have also been attempted (e.g. Kinney & Brisudova 2000), which also suffer from fatal flaws (as they noted), for example the problem with explaining gravitational lensing. MOND needs to appeal to some as yet unknown process which reproduces the usual 'GR factor of 2' in order to explain lensing results (Qin et al. 1995) for astrophysical objects like the sun, as well as for cosmological lensing . For objects we would classically consider dark matter dominated, the situation is much worse. Consider, for example, clusters of galaxies, whose mass estimated from dynamical arguments and gravitational lensing approximately agree if $\Omega_{\rm CDM} \simeq 10 \times \Omega_{\rm B}$. If light doesn't couple to gravity the same way as baryons, then you have a factor of ~ 10 discrepancy.

Zhitnikov & Nester (1994) presented a very elegant argument about the form of reasonable extensions to GR which might explain flat rotation curves. They assumed that such modifications are in the framework of metric theories which also conform to several other rather weak assumptions. The most general metric that they found is more general than the usual Parameterized Post-Newtonian formalism (see e.g. Will 1993). They found that constraints on the terms and symmetries within such a metric (coming from the necessity to fit with solar system experiments, gravitational deflection of light, etc.) make it seem rather improbable that flat rotation curves can be explained away without the need for dark matter.

MOND breaks several of the assumptions made by Zhitnikov & Nester (1994). However, the proponents of MOND suggest no other consistent framework with which to replace the conventional picture. This makes it currently impossible to carry out definitive cosmological calculations in a MONDian Universe.

7   CONCLUSIONS

Recently, there has been a renewed interest in the dark matter sector of the standard cosmological theory. In the absence of direct detection of a dark matter candidate, this is understandable. Some authors have even revived the idea

that a plausible explanation of galaxy rotation curves, one of the many pieces of evidence for dark matter, lies not in the gravitational influence of non-luminous matter but in a modification of our fundamental law of gravity. Perhaps the longest lived such alternative is that of Milgrom, who proposed the MOdified Newtonian Dynamics.

We have reviewed the extensive literature on MOND, including many studies which show that MOND is both a *drastic* modification of our laws of motion and gravity *and* that it fares poorly in explaining extant observations of galactic structure. We have included these since it is difficult to find a comprehensive discussion of these issues published anywhere. The focus of our paper however is on the recent attempts to extend MOND to a theory of cosmology.

We consider the numerous conceptual problems inherent in this approach, which make MONDian cosmology much less than a theory. We believe that the recent claims of MONDian 'predictions' or 'cosmological models' are unsupportable. Within the current MOND framework, the relevant calculations are fraught with ambiguities and inconsistencies, making any quantitative calculations impossible.

In our opinion, the entire premise of MONDian cosmology is at odds with the modern view of the formation and evolution of galaxies. The MOND calculations are deeply rooted in the archaic models of monolithic galaxy formation, with galaxies being unchanging, isolated, eternal objects whose centres define special places in the universe about which accelerations can be measured. It is well nigh impossible to embed this theory within the modern context wherein galaxies are interacting, dynamic objects, part of the evolving large-scale structure of the universe. MOND is at odds with the simplicity of hierarchical clustering through gravitational instability which is a well-tested and highly successful paradigm. In this picture the hot early universe was a simpler place, for which calculations can be done with great precision.

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